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On the Possibility of Abnormally Intense Radiation Due to the Rotation of Electron Around a Dielectric Sphere

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Abstract
The abnormally intense radiation due to the uniform rotation of electron around the equatorial plane of a dielectric sphere is obtained. It takes place when the sphere surface is at a specific distance from the electron orbit and when the Cherenkov condition for electron and the matter of the sphere is satisfied.

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1 Introduction

A number of important electromagnetic processes is conditioned by the matter: the Vavilov-Cherenkov radiation, the X-ray transition radiation, the radiation of channeled particles [1] - [9]. In this connection it is of interest to study an influence of the matter on the radiation of the relativistic charge rotating along a circle in a permanent magnetic field (synchrotron radiation [10, 11]).

The synchrotron radiation in an infinite uniform medium was studied in [12] and further in [2, 13]. The radiation of a nonrelativistic particle rotating uniformly around a dielectric sphere, and the radiation of the particle rotating in close proximity to the ideally conducting sphere were considered in the [14]. In [15, 16] the expressions were obtained for the spectral and spectral-angular distribution of the radiation intensity without restrictions on the orbit radius and velocity of a particle rotating around a sphere with an arbitrary dielectric permittivity.

In the present paper an analysis of the numerical calculations by the formulae obtained in [15, 16] is carried out. The peculiarities of the radiation conditioned by the matter of a sphere and by its size, are revealed.

2 Basic formulae

We present the basic formulae describing the radiation of a particle with the charge q and velocity $v = \omega_e r_e$ uniformly rotating around a sphere in its equatorial plane (r_e is the radius of orbit). The magnetic permeability of the sphere we take equal to unity and consider its dielectric permittivity ε_0 as an arbitrary real quantity (we do not take into account the effects connected with the radiation absorption), the sphere radius $r_o < r_e$. The radiation

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intensity at the frequency $\omega = k\omega_e$ (after an averaging over the rotation period $2\pi/\omega_e$) is determined by the expression

$$I_k = 2\frac{q^2\omega_e^2}{c\sqrt{\varepsilon_1}} \sum_{s=0}^{\infty} (\mid a_{kE}(s) \mid^2 + \mid a_{kH}(s) \mid^2), \tag{1}$$

where ε_1 is the dielectric permittivity of a medium surrounded the sphere,

$$a_{kE} = k b_l(E) P_l^k(0) \sqrt{\frac{(l-k)!}{l(l+1)(2l+1)(l+k)!}}, \qquad l = k + 2s,$$

$$a_{kH} = b_l(H) \sqrt{\frac{(2l+1)(l-k)!}{l(l+1)(l+k)!}} \cdot \frac{dP_l^k(y)}{dy}, \qquad y = 0, \qquad l = k + 2s + 1 \tag{2}$$

are the dimensionless amplitudes describing the contributions of multipole of the electric and magnetic kinds, respectively. In Eq.(2) $P_l^k(y)$ are the associated Legendre polynomials, and b_l is a factor depending on k, $x = r_0/r_e$, ε_0 and ε_1:

$$b_l(H) = iu_1 \left[j_l(u_1) - h_l(u_1) \frac{\{j_l(\underline{x}u_0), j_l(\underline{x}u_1)\}}{j_l(xu_0)h_l(xu_1)} \right], \qquad u_i = k\sqrt{\varepsilon_i}\frac{v}{c},$$

$$b_l(E) = (l+1)b_{l-1}(H) - l b_{l+1}(H) + \frac{1}{x^2}\left(\frac{1}{\varepsilon_0} - \frac{1}{\varepsilon_1}\right) \times$$

$$\times \left[j_{\underline{l-1}}(xu_0) + j_{\underline{l+1}}(xu_0) \right] \left[h_{\underline{l-1}}(u_1) + h_{\underline{l+1}}(u_1) \right] \frac{l(l+1)u_0 j_l(xu_0)}{l z_{l-1}^l + (l+1) z_{l+1}^l}, \tag{3}$$

where $h_l(y) = j_l(y) + in_l(y)$; j_l and n_l are the spherical Bessel and Neumann functions, respectively. In Eq.(3) the following notations are introduced:

$$\{a(\underline{x}u_i), b(\underline{x}u_j)\} = a \cdot \frac{\partial b}{\partial x} - \frac{\partial a}{\partial x} \cdot b, \qquad f_{\underline{l}}(y) = \frac{f_l(y)}{\{j_l(\underline{x}u_0), h_l(\underline{x}u_1)\}},$$

$$z_\nu^l = \frac{u_1 j_\nu(xu_0) h_l(xu_1)/\varepsilon_1 - u_0 j_l(xu_0) h_\nu(xu_1)/\varepsilon_0}{u_1 j_\nu(xu_0) h_l(xu_1) - u_0 j_l(xu_0) h_\nu(xu_1)}. \tag{4}$$

The derivation of Eq.(1) is given in [15, 16].

In the case of homogeneous medium ($\varepsilon_0 = \varepsilon_1 = \varepsilon$)

$$b_l(H) = iuj_l(u), \qquad u = k\sqrt{\varepsilon}\frac{v}{c},$$

$$b_l(E) = iu(2l+1)\left[j_l'(u) + \frac{1}{u} j_l(u) \right], \tag{5}$$

and therefore Eq.(1), naturally, does not depend on x. One can also be convinced that Eq.(1) is transformed into the known formula [2, 10, 12, 13, 17]

$$I_k = kvq^2\frac{\omega_e^2}{c^2}\left[2J_{2k}'(2k\beta\sqrt{\varepsilon}) + (1 - \frac{1}{\varepsilon\beta^2}) \int_0^{2k\beta\sqrt{\varepsilon}} J_{2k}(y)dy \right], \tag{6}$$

where $\beta = v/c$, $J_k(y)$ is the integer-order Bessel function, and $\varphi'(y) = d\varphi/dy$.

3 Results of numerical calculations

In Fig.1 along the axis of ordinates we plotted an average number of electromagnetic field quanta

$$n_k = \frac{2\pi I_k}{k\hbar\omega_e^2}, \tag{7}$$

radiated per one period of rotation of electron with the energy 2 MeV (the logarithmic scale), and along the axis of abscissa an order of radiated harmonic in the range $1 \leq k \leq 50$ is plotted. The function n_k is presented for the four values of x. The curves a, b, c, d are the polygonal lines connecting the points with different k and the same x_a, x_b, x_c and x_d, respectively. The line a describes a rotation in vacuum ($x_a = 0$), and the line b describes a rotation in the continuous medium ($x_b = \infty$) with the dielectric permittivity $\varepsilon = 3$ (the Cherenkov condition is satisfied). The calculations were carried out by the formula (6). For simplicity the dependence of ε on k (the dispersion) is not taken into account. It followed from the plots that in a continuous media

$$n_k(\infty) \leq \frac{ve^2}{hc^2}\left(1 - \frac{1}{\varepsilon\beta^2}\right) < \frac{e^2}{hc} \approx 0.05 \tag{8}$$

is larger than the analogous quantity $n_k(0)$ in the empty space. A difference between $n_k(\infty)$ and $n_k(0)$ is conditioned by the contribution of the Cherenkov's quanta. Along with this, the specific oscillations [12] are revealed on the curve b. They results from the interference of waves in the conditions when the velocity of the electromagnetic waves propagation is lower than the velocity of the source motion $c/\sqrt{\varepsilon} < v$.

A similar pattern should be observed also in the case when a medium has finite sizes. In the section 2 we considered the case of a sphere with the radius r_o, around of which electron rotates at the distance $r_e - r_o$. The polygonal lines c and d represent the results of calculations by the formula (1) for the two fixed values $r_o/r_e = 0.974733692 = x_c$ and $0.980861592 = x_d$, respectively. The dielectric permittivity of the sphere $\varepsilon_0 = 3$. Outside the sphere there is a vacuum ($\varepsilon_1 = 1$). The electron energy $E_e = 2MeV$. As it is seen, the specific oscillations are observed also in this case. However, there are also the peaks, and on the corresponding harmonics ($k = 26$ for the case c and $k = 40$ for the case d) the radiation is abnormally intensive:

$$\begin{aligned} n_{26}(x_c) &= 4300 \qquad \text{for the curve } c\,, \\ n_{40}(x_d) &= 94 \qquad \text{for the curve } d. \end{aligned} \tag{9}$$

At the same time on the neighbouring harmonics $n_k(x)$ is of the order $n_k(\infty)$.

In the empty space the radiation intensity I_k reaches a maximum on the harmonic with $k_{max} = 26$: [2] $I_{26}(0) = 0.96e^2\omega_e^2/c$. On this harmonic an influence of the sphere with the radius $r_o = 0.974733692\ r_e$ is the most intensive: $I_{26}(x_c)/I_{26}(0) \approx 2.53 \cdot 10^6$ (just this value of r_o is chosen in the case of the curve c). An analogous situation is possible also on other harmonics. For example, on the harmonic with $k = 40$ an influence of the sphere is maximal at $r_o = 0.980861592\ r_e$ (the curve d). In this case $I_{40}(x_d)/I_{26}(0) \approx 55700$.

Figs.2 and 3 show the dependence of $n_k(x)$ on x for the harmonics with $k = 26$ and $k = 40$, respectively. In this plots also $\varepsilon_0 = 3$, $\varepsilon_1 = 1$ and $E_e = 2MeV$. Against a

[2] This result is obtained also from the formula $k_{max} = 0.44(E_e/m_ec^2)^3$ which is valid for ultrarelativistic electron [17].

background of the oscillations of the function $n_k(x)$, the extremely narrow and very high peaks are observed (on the right-hand part the function $n_k(x)$ is shown in the vicinity of the maximal peak). Already at a small deviation (along the axis of abscissa) from the centre of any of these peaks n_k rapidly decreases. Therefore the value $x = r_o/r_e$ must be fixed with a high accuracy (for example, by an external electric field sustaining a uniform rotation of a particle). The energy radiated per one period of the electron rotation, is equal to

$$\frac{2\pi}{\omega_e} I_k = k\hbar\omega_e n_k. \tag{10}$$

The radiative losses are negligible if the cyclic frequency

$$\omega_e \ll \frac{E_e}{k\hbar n_k} \sim 10^{13} \frac{E_e}{MeV} \frac{10^8}{kn_k} Hz. \tag{11}$$

An analogous pattern takes place for other $1 < \varepsilon_0 \leq 5$ and $E_e \leq 5MeV$, when the Cherenkov condition is satisfied (see Table1). Moreover, in certain cases (see the 2-4th rows of Table 1) one can observe a superintensive radiation with

$$n_k > \frac{2\pi r_e}{\lambda_k} = k\frac{v}{c}. \tag{12}$$

Table 1: The average number n_k of electromagnetic field quanta emitted per revolution of electron.

		Rotation in a continuos medium			Rotation around a sphere in a vacuum			
k	E_e	$\varepsilon = 1$	$\varepsilon = 3$	$\varepsilon = 5$	$\varepsilon = 3$		$\varepsilon = 5$	
	MeV	n_k	n_k	n_k	μ	$n_k(\mu)$	μ	$n_k(\mu)$
	1	$3.07 \cdot 10^{-4}$	$2.37 \cdot 10^{-2}$	$3.18 \cdot 10^{-2}$	6.6433228	4.13	5.2992	1.76
20	3	$2.72 \cdot 10^{-3}$	$3.32 \cdot 10^{-2}$	$3.33 \cdot 10^{-2}$	0.5432354	201	3.482	0.34
	5	$3.00 \cdot 10^{-3}$	$3.42 \cdot 10^{-2}$	$3.63 \cdot 10^{-2}$	1.480803	133	2.596109	133
	1	$2.39 \cdot 10^{-5}$	$1.93 \cdot 10^{-2}$	$3.11 \cdot 10^{-2}$	0.82132	9.64	1.13910742	2260
40	3	$1.57 \cdot 10^{-3}$	$2.90 \cdot 10^{-2}$	$3.77 \cdot 10^{-2}$	1.2224	0.65	0.9986	0.65
	5	$1.85 \cdot 10^{-3}$	$3.22 \cdot 10^{-2}$	$3.47 \cdot 10^{-2}$	4.801	0.16	1.50036	1.45

Note: ε is the dielectric permittivity of the matter. In the case of a sphere for every three values of k, E_e and ε we chosed and presented one value of the ratio of the sphere radius to the radius of the electron orbit $r_o/r_e = 1 - 0.01\mu$, for which $n_k(\mu)$ is considerably larger than e^2/hc.

The formulae (3) are not valid for electron rotating inside a spherical cavity in an infinite medium, and therefore we did not carry out the corresponding calculations.

The numerical calculations were duplicated by two independent programs. One of them, a more simple, was made with the help of the Mathematica, and an another, more fast-acting, on the Pascal language.

4 Conclusions

We calculated the intensity of radiation for electron with an energy of several MeV uniformly rotating around a sphere in its equatorial plane. The matter of the sphere is regarded as transparent, and its dielectric permittivity $1 < \varepsilon \leq 5$. It is obtained that on the average the $n > k$ quanta of the electromagnetic field may be radiated per revolution of electron, where k is the number of the radiated harmonic ($k \leq 50$). In the absence of a sphere or at the rotation of electron in an infinite medium with the same ε, the analogous quantity $n_k < 0.05 \approx e^2/hc$. Such an intense radiation takes place when the sphere surface is at a specific distance from the electron orbit and when the Cherenkov condition for electron and the matter of the sphere is satisfied.

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Figure captions:

Fig.1: Average number $n_k(x)$ of electromagnetic field quanta emitted per revolution of electron, as a function of the radiated harmonic's number k. The polygonal lines a, b, c and d differ by the value of x (the ratio of the sphere radius to the radius of the electron orbit): $x_a = 0$ (vacuum), $x_b = \infty$ (infinite medium), $x_c \approx 0.9747337$, $x_d \approx 0.9808616$. The dielectric permittivity of the matter $\varepsilon = 3$, the electron energy $E_e = 2MeV$.

Fig.2: The same quantity, as in Fig.1, depending on x. A number of the radiated harmonic is fixed: $k = 26$. Here also $\varepsilon = 3$ and $E_e = 2MeV$. On the right-hand side the function $n_k(x)$ is plotted in the vicinity of the maximal peak.

Fig.3: The same dependence, as in Fig.2, in the case $k = 40$.

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